

Quasiparticles in the Vortex State of Dirty d -Wave Superconductors

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We consider the problem of the vortex contribution to thermal properties of dirty d -wave superconductors. In the clean limit, Volovik has argued that the main contribution to the density of states in a d -wave superconductor arises from extended quasiparticle states which may be treated semiclassically, giving rise to a specific heat contribution $\delta C(H) \lesssim H^{1/2}$. We show that the extended states continue to dominate the dirty limit, but lead to a $H \log H$ behavior at the lowest fields, $H_{c1} \ll H \ll H_{c2}$. This crossover may explain recent discrepancies in specific heat measurements at low temperatures and fields in the cuprate superconductors. We further discuss the field dependence of transport properties within the same model.

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Introduction. As pointed out by Volovik, [1] d -wave superconductors are fundamentally different from their s -wave counterparts in the vortex state, in that at low temperatures and fields the extended quasiparticle states around the vortex play a much more important role. In the case of the single-particle density of states, he showed in fact that the contribution from these states over the entire vortex unit cell exceeds that of the bound states. It is therefore to be expected that the quasiparticles will also play an important role in transport properties of a d -wave system in the presence of an applied magnetic field. To study low- T transport, one needs a way of including scattering by defects at a simple level. We recently studied the specific heat $\delta C(H)$ of $d_{x^2-y^2}$ superconductors in a field using the common t -matrix approximation for strong impurity scattering, [2] proposing a possible explanation for deviations of some measurements from the $\delta C \sim \sqrt{H}$ behavior predicted for a clean d -wave superconductor by Volovik. We also showed how other consequences of the clean Dirac spectrum, including the $1/\omega$ divergence in the density of states $\delta N(\omega; H)$ [8] and the predicted scaling [3] of the specific heat $\delta C/(TH^{1/2}) = F_C(H^{1/2}/T)$, are modified by disorder.

In this paper we review the results on thermodynamic properties and present a preliminary calculation of the low-field microwave conductivity. We neglect other possible contributions to the conductivity arising from disordered vortex scattering, and assume that the Abrikosov lattice is pinned, ordered, and sufficiently dilute to not alter the quasiparticle bands significantly from their 1-vortex form. We point out that confirmation of the magnetic field and disorder dependence would not only provide evidence for the primacy of quasiparticle transport in d -wave superconductors, but also yield information on the phase shifts of the defects and on quasiparticle relaxation times.

Semiclassical treatment of extended states. In the semiclassical approximation the Doppler effect associated with supercurrents around a vortex lead to a shift of the quasiparticle energy $\omega \rightarrow \omega - \mathbf{v}_s \cdot \mathbf{k}$, where $\mathbf{v}_s = (\hbar/2mr)\hat{\theta}$

is the superfluid velocity. The impurity-averaged electron propagator in particle-hole space is given by

$$g(\mathbf{k}, \omega; \mathbf{v}_s) = \frac{(\tilde{\omega} - \mathbf{v}_s \cdot \mathbf{k})\tau_0 + \Delta_{\mathbf{k}}\tau_1 + \xi_{\mathbf{k}}\tau_3}{(\tilde{\omega} - \mathbf{v}_s \cdot \mathbf{k})^2 - \Delta_{\mathbf{k}}^2 - \xi_{\mathbf{k}}^2}, \quad (1)$$

where the τ_i are Pauli matrices. Due to symmetry of the $d_{x^2-y^2}$ order parameter $\Delta_{\mathbf{k}} = \Delta_0 \cos 2\phi$ and the assumed particle-hole symmetry of the normal state only the frequency is renormalized $\tilde{\omega} = \omega - \Sigma_0(\omega)$. In the unitarity limit the self-energy is given by $\Sigma_0 = \tau_0 \Gamma / G_0$ where $\Gamma = n_i / \pi N_0$ is an impurity scattering rate depending on the concentration n_i of point potential scatterers and the density of states at the Fermi level, N_0 . For a simple $d_{x^2-y^2}$ superconductor the averaged integrated Green's function reads $G_0(\omega, \mathbf{v}_s) = -i(2/\pi)\mathbf{K}(\Delta_0/(\tilde{\omega} - \mathbf{v}_s \cdot \mathbf{k}))$, where \mathbf{K} is the complete elliptic integral of the first kind.

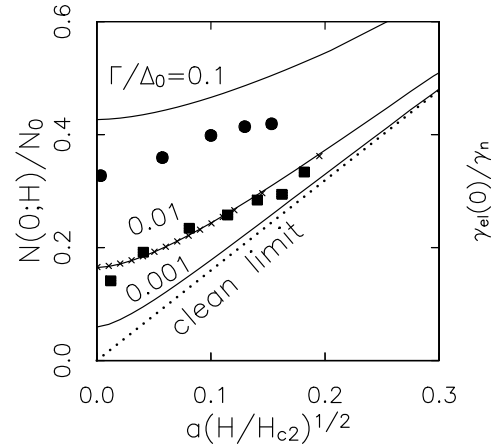


FIG. 1. $N(0; H)/N_0$ for $\Gamma/\Delta_0 = 0.1, 0.01$, and 0.001 in unitarity limit (solid lines) and the clean limit (dotted line). Data from Fisher et al. [6] (circles); Moler et al. [4] (untwinned sample, squares), assuming $H_{c2}/a^2 = 300\text{T}$, $\gamma_n = 15 \text{ mJ-mol-K}^2$.

Density of states at zero energy. To find $N(0; H)$, we average the propagator over a vortex unit cell, $N(0; H)/N_0 \equiv \langle -\text{Im}G_0(\omega; \mathbf{v}_s) \rangle_H$, where for any $f(\mathbf{v}_s)$ we define $\langle f(\mathbf{v}_s) \rangle_H \equiv A^{-1}(H) \int_{\text{cell}} d^2r f(\mathbf{v}_s)$. The magnetic field dependence enters only through the inter-vortex spacing $R = \xi_0(\pi/2)^{1/2}a^{-1}(H_{c2}/H)^{1/2}$. In

the clean limit we reproduce the result of Volovik $N(0; H)/N_0 \simeq \sqrt{8/\pi} a(H/H_{c2})^{1/2}$. In the dirty limit, where the zero-energy quasiparticle scattering rate γ_0 is much larger than the average quasiparticle energy shift $E_H \equiv a(H/H_{c2})^{1/2} \Delta_0$, we find

$$\frac{\delta N(0, H)}{N_0} \simeq \frac{\Delta_0}{8\gamma_0} a^2 \left(\frac{H}{H_{c2}} \right) \log \left[\frac{\pi}{2a^2} \left(\frac{H_{c2}}{H} \right) \right] \quad (2)$$

Numerical results for the density of states $N(0, H)$ together with experimental data of Moler et al. [4] and Fisher et al. [6] are given in Fig. 1. The data Moler et al. are consistent with a slightly dirty d-wave superconductor, while the Fisher data cannot be well fitted to the clean case, suggesting that their sample contains roughly an order of magnitude more defects.

Density of states at finite frequency. The field dependent part of the density of states for $v_s k_F \ll \omega \ll \Delta_0$ is given by $\langle -\text{Im}[G_0(\omega; \mathbf{v}_s) - G_0(\omega)] \rangle_H$ [8] which for $\gamma_0, E_H < \omega$ yields

$$\frac{\delta N(\omega; H)}{N_0} \simeq \left\langle \left| \frac{\mathbf{v}_s \cdot \mathbf{k}_n - \omega}{\Delta_0} \right| - \left| \frac{\omega}{\Delta_0} \right| \right\rangle_H \simeq \frac{E_H}{\sqrt{2\pi} \Delta_0^2} F(x) \quad (3)$$

$$F(x) \equiv \frac{1}{x} \begin{cases} \pi/2 & x > 1 \\ 3x\sqrt{1-x^2} + (1+2x^2)\sin^{-1}x - \pi x^2 & x < 1 \end{cases}$$

where $x = \sqrt{2/\pi}(\omega/E_H)$ and \mathbf{k}_n the nodal direction.

In Fig. 2 we plot the density of states versus frequency. In the intermediate range we recover the predicted $1/\omega$ divergence $\delta N(\omega; H)/N_0 \simeq a^2 \pi \Delta_0 H / (4\omega H_{c2})$, which in the clean limit $\gamma_0 < E_H$ is cut off as $\delta N(\omega; H) \simeq N(0; H)(1 - \pi x/4)$ and in the dirty case by the impurity scattering scale $\gamma_0 > E_H$.

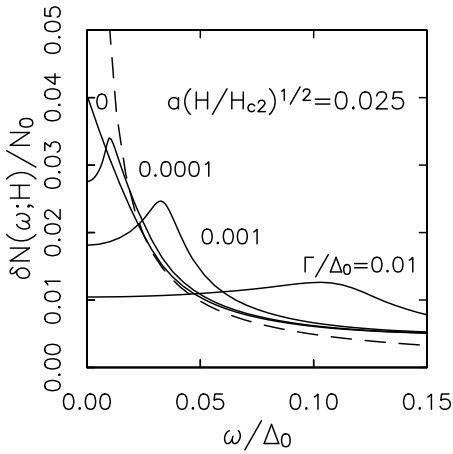


FIG. 2. Density of states at $a(H/H_{c2})^{1/2} = 0.025$ vs. freq. ω/Δ_0 for $\Gamma/T_c = 0, 0.0001, 0.001, 0.01$ (solid lines). Intermediate-frequency asymptotic result $\delta N \simeq a^2 \pi \Delta_0 H / (4\omega H_{c2})$ (dashed line).

Scaling of specific heat. The specific heat at low-temperature is given by

$$C \simeq 2 \int_0^\infty d\omega \left(\frac{\omega}{T} \right)^2 \left(\frac{-\partial f}{\partial \omega} \right) N(\omega; H) \quad (4)$$

$$\simeq \begin{cases} N(0; H) \frac{\pi^2}{3} T & T \ll \max[\gamma_0, E_H] \ll \Delta_0 \\ N_0 \left(\frac{9\zeta(3)T^2}{\Delta_0} \right) & \gamma_0, E_H \ll T \ll \Delta_0 \end{cases}$$

A T^2 term characteristic of the pure d-wave system in zero field is present whenever both the impurity and magnetic field scales are smaller than the temperature. Substituting Eq. (3) into (4) leads to the scaling function of the specific heat $\delta C(H)/[\gamma_n T a(H/H_{c2})^{1/2}] = F_C[Y]$, where $Y = a(H/H_{c2})^{1/2} T_c/T$. The scaling function F_C varies as $F_C \simeq 3 \log 2 \Delta_0 Y / (4\pi T_c)$ for $Y \ll 1$ and as $F_C \simeq \sqrt{2/\pi}$ for $Y \gg 1$. Scaling is expected for a given data set provided H, T are such that E_H and T are both larger than the impurity scale γ_0 . A full numerical evaluation of (4) plotted in Fig. 3 shows that for the clean case (open symbols), scaling is obtained over the full range of Y , whereas for the dirty system (filled symbols) scaling has broken down completely.

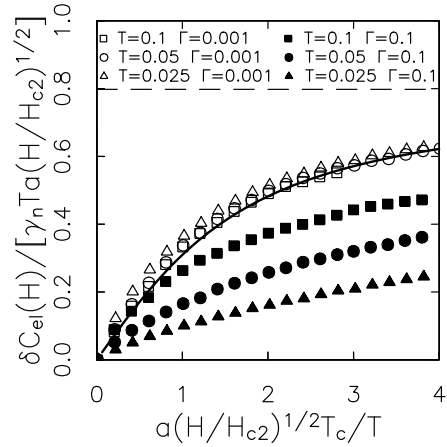


FIG. 3. Normalized vortex contribution to specific heat, $\delta C_{el}(H)/[\gamma_{el} T a(H/H_{c2})^{1/2}]$ vs. $Y \equiv a(H/H_{c2})^{1/2} T_c/T$ for fixed temperatures T and scattering rates Γ as shown; unit of energy T_{c0} . Asymptotic large- Y limit $\sqrt{2/\pi}$ (dashed line).

Microwave conductivity. The microwave conductivity is given by [9]

$$\sigma_{ij}(\Omega) = -\frac{ne^2}{m} \int_{-\infty}^{+\infty} d\omega \left[\frac{\beta}{4} \text{sech}^2\left(\frac{\beta\omega}{2}\right) \cdot S_{ij}(\omega, \Omega) \right], \quad (5)$$

where

$$S_{ij}(\omega, \Omega) = \text{Im} \int \frac{d\phi}{2\pi} \hat{k}_i \hat{k}_j \cdot \left[\frac{\tilde{\omega}'_+}{\tilde{\omega}_+ - \tilde{\omega}'_+} \left(\frac{1}{\xi'_{0+}} - \frac{1}{\xi_{0+}} \right) + \frac{\tilde{\omega}'_-}{\tilde{\omega}_+ - \tilde{\omega}'_-} \left(\frac{1}{\xi_{0+}} + \frac{1}{\xi'_{0-}} \right) \right]. \quad (6)$$

Here we have defined $\tilde{\omega}_{\pm} = \bar{\omega} - \Sigma_0(\omega \pm i0^+)$, $\xi_{0\pm} = \pm \text{sgn}(\bar{\omega})(\bar{\omega}_{\pm}^2 - \Delta_k^2)^{1/2}$, as well as analogous primed quantities $\tilde{\omega}'_{\pm} = \tilde{\omega}_{\pm}(\bar{\omega} - \Omega)$ and $\xi'_{0\pm} = \xi_{0\pm}(\bar{\omega} - \Omega)$, with $\bar{\omega} = \omega - \mathbf{v}_s \cdot \mathbf{k}_n$. Taking the limit $\Omega \rightarrow 0$, $T \rightarrow 0$ and

performing the ϕ -integration we find for the contribution to the conductivity from currents at a position \mathbf{r} relative to the vortex center

$$S_{ii}(\omega, \mathbf{r}) = \frac{1}{4} \sum_{nodes} \frac{1}{\pi \Delta_0} \text{Re} \left\{ k \left[\frac{\bar{\omega}}{\Sigma_0} \mathbf{K}(k) - \mathbf{E}(k) \right] \right\}, \quad (7)$$

where $\mathbf{K}(k)$ and $\mathbf{E}(k)$ are the complete elliptic integrals of the first and second kind, respectively, with the argument $k = \Delta_0 / (\Delta_0^2 - (\Sigma_0 - \bar{\omega})^2)^{1/2}$. Note the space dependence arises exclusively through the Doppler shift $\mathbf{v}_s \cdot \mathbf{k}_n = (\hbar v_F / 2r) \cos \theta$. At low temperature and for $H_{c1} \ll H \ll H_{c2}$ the leading order is given by

$$\begin{aligned} \sigma(\mathbf{v}_s) = & \frac{\sigma_{00}}{4} \sum_{nodes} \int_{-\infty}^{+\infty} d\omega \frac{\beta}{4} \text{sech}^2\left(\frac{\beta\omega}{2}\right) \\ & \cdot \left\{ 1 + \left[\left(\frac{\Sigma_0'' \bar{\omega}}{\Delta_0^2} \right) + \left(\frac{\bar{\omega}}{\Sigma_0''} \right) \right] \arctan \left(\frac{\bar{\omega}}{\Sigma_0''} \right) \right. \\ & \left. + \frac{1}{2} \left(\frac{\Sigma_0''^2 - \bar{\omega}^2}{\Delta_0^2} \right) \left[\ln \left(\frac{4\Delta_0}{\sqrt{\Sigma_0''^2 + \bar{\omega}^2}} \right) - \frac{3}{2} \right] \right\}, \quad (8) \end{aligned}$$

where $\sigma_{00} = \frac{ne^2}{m} \frac{1}{\pi \Delta_0}$ is the universal conductivity [10]. In the gapless regime the self-energy reduces to $-\Sigma_0'' \sim \gamma_0 \gg E_H$ which leads to the following leading order contribution of the magnetic field to the conductivity

$$\delta\sigma(\mathbf{r}) = \sigma_{00} \left(\frac{\mathbf{v}_s \cdot \mathbf{k}_n}{\gamma_0} \right)^2. \quad (9)$$

In the clean limit $\gamma_0 \ll E_H$ the self-energy $\Sigma_0(\omega = 0, \mathbf{r})$ is given by

$$\Sigma_0''(\omega = 0, \mathbf{r}) \sim -\Gamma \left| \frac{\Delta_0}{\mathbf{v}_s \cdot \mathbf{k}_n} \right| \quad (10)$$

which leads to

$$\delta\sigma(\mathbf{r}) = \sigma_{00} \frac{\pi}{2} \left(\frac{\mathbf{v}_s \cdot \mathbf{k}_n}{\Delta_0} \right)^2 \frac{\Delta_0}{\Gamma}. \quad (11)$$

By averaging the above results over a unit vortex cell we obtain the magnetic field contribution to the zero-frequency conductivity for a d-wave superconductor

$$\frac{\delta\sigma}{\sigma_{00}} = \begin{cases} \frac{\pi}{4} \left(\frac{\Delta_0}{\gamma_0} \right)^2 a^2 \left(\frac{H}{H_{c2}} \right) \ln \left(\frac{\pi}{2a^2} \frac{H_{c2}}{H} \right) & \gamma_0 \gg E_H \\ \frac{\pi^2}{8} \frac{\Delta_0}{\Gamma} a^2 \left(\frac{H}{H_{c2}} \right) \ln \left(\frac{\pi}{2a^2} \frac{H_{c2}}{H} \right) & \gamma_0 \ll E_H \end{cases} \quad (12)$$

These results should be valid provided the quasiparticle mean free path is in fact limited by impurities and not by fluctuations in the vortex lattice. The experiments of Orenstein et al. [12] at 150 GHz and a few Tesla are in precisely the correct regime to allow one to neglect absorption into vortex oscillation modes, and to compare to our $\Omega \rightarrow 0$ result. Furthermore, we expect even in the presence of elastic (pinned) vortex lattice disorder

that for small fields of order a few Tesla, vortices will be sufficiently dilute that scattering at low T will be impurity dominated. Orenstein et al. indeed observe a convex downward curvature in $\delta\sigma(H)$, as well as a broad maximum at around 3T. Further investigation of scattering of quasiparticles by vortices and of the effect of applied field on inelastic spin fluctuations is needed to extend our results to higher temperatures and fields.

Note added: Results for $N(0)$ in the dirty limit similar to ours were obtained independently by Barash et al. [13]

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